Virtual galaxy formation

August E. Evrard

Phil. Trans. R. Soc. Lond. A 2000 **358**, 2143-2152
doi: 10.1098/rsta.2000.0636

10.1098/rsta.2000.0636

Virtual galaxy formation

By August E. Evrard

Department of Physics, University of Michigan, Ann Arbor, MI 48109-1120, USA

In this contribution, I briefly review the status of efforts to directly simulate the evolution of galaxies and their pre- and post-formation environments. A few phenomenological applications are discussed: the intracluster medium in galaxy clusters, the structure of the Lyα forest, galaxy clustering in cosmic volumes and the internal structure of galaxies. Given the strongly empirical character of the problem, 'reverse engineering' of the observed galaxy distribution is a fair characterization of the near-term state of affairs. The thorny problem of how galaxies trace the overall mass on large scales may be yielding some ground, but efforts to resolve smaller-scale features of galaxies are challenged by the complex nature of star–gas interactions on kiloparsec scales.

Keywords: cosmology; galaxy formation; numerical simulation

1. Introduction

Modelling galaxy formation in a cosmic environment is a formidable task for a number of reasons. For starters, there is the problem of dynamic range. The characteristic half-light radii of bright galaxies are three orders of magnitude smaller than their mean separation, implying a range of 10^9 in volume. A similarly large factor separates the stellar mass of an L_* galaxy from the mass of the smallest star forming region within it. Second, the complexity of the physical processes driving the conversion between gas and stars makes it difficult to justify a particular *a priori* description employed in numerical modelling. Finally, the problem is intrinsically three-dimensional; there are no spatial symmetries to exploit.

These factors are good excuses for pessimism, but nature has kindly provided us with reasons to be optimistic by giving us galaxy populations with remarkable structural regularities. There is hope, then, to reverse engineer the process. The morphological classes of the Hubble sequence, the Tully–Fisher relation for disc galaxies, the fundamental plane of elliptical galaxies, and the morphology–density relation in clusters imply the existence of organizing principles that we can hope to reproduce by modelling what might be termed 'mesoscopic' phenomena. Essentially, the problem boils down to identifying one or more 'recipes' for the star formation rate as a function of local conditions and for the feedback, in terms of thermal, chemical and kinetic returns, which that star formation has on its local environment.

So-called semi-analytic methods (SAMs), reviewed by Cole (this issue), offer an alternative approach to direct dynamical modelling. SAMs offer the ability to explore the controlling parameter space far more quickly than would be possible via direct simulation, but the approximations made to describe the dynamical behaviour of the galaxies can be a liability for these methods, to a degree depending on the

Phil. Trans. R. Soc. Lond. A (2000) **358**, 2143–2152

particular issues under investigation. Clearly, the way forward is to combine both approaches. Full dynamical modelling can assess the accuracy of approximations employed by SAMs and the SAMs, in turn, can provide best estimate prescriptions for star formation and feedback to be used in the full dynamical modelling.

2. Engineering galaxies: processes and limitations

In a universe with matter content dominated by cold dark matter, the basic physical ingredients for galaxy formation outlined in figure 1 are now familiar. Gravity is, of course, the key player. The growth of density fluctuations into the nonlinear regime leads to the emergence of a hierarchically evolving cosmic web of collapsed, dark matter halos. Within the evolving potential well of a given halo, the baryonic matter is heated by shocks but the optically thin gas can rapidly lose its thermal pressure support via radiative cooling. The baryons then sink toward the halo core and, once self-gravitating, suffer fragmentation instabilities that produce local star forming regions. The star formation process is represented as a black box in figure 1 because it is practically impossible to describe from first principles. Instead, one employs an empirically or heuristically motivated prescription. For example, a formulation commonly used since Larson (1974) assumes that a fixed fraction of gas is consumed in star formation over a local dynamical time $\tau_{\mathrm{dyn}} \propto (G\rho)^{-1/2}$:

$$\frac{\mathrm{d}\rho_*}{\mathrm{d}t} = c_* \frac{\rho_{\mathrm{gas}}}{\tau_{\mathrm{dyn}}}. \tag{2.1}$$

Modelling feedback from star formation is accomplished by introducing a stellar mass function and calculating the mass and energy return over time from stellar winds and supernovae. An important and presently uncertain issue is the question of the relative amounts of thermal and kinetic energy returned from star formation (Katz 1992; Navarro & White 1994). Since feedback is believed to be a dominant player only in the smallest halos, its effects are often neglected in simulations designed to model the formation of bright galaxies in large cosmic volumes (Katz *et al*. 1992; Evrard *et al*. 1994). The lack of small-scale resolution in these experiments serves to mimic feedback, at least in the sense that gas cannot be converted into stars on unresolved scales.

Given prescriptions for star formation processes, the equations describing the combination of ingredients in figure 1 constitute a well-posed initial-value problem that can be solved within a finite domain with appropriate boundary conditions. Bertschinger (1998) presents the governing equations (see also Cen 1992) and reviews various choices of numerical methods. Eulerian (decomposition by volume) and Lagrangian (decomposition by mass) approaches offer complementary advantages. Broadly speaking, advantages of Eulerian schemes are better shock capturing, calculable truncation error, and equal spatial resolution of low- and high-density regions. Advantages of Lagrangian schemes are automatic advection of fluid properties, enhanced spatial resolution in high-density regions (for schemes that scale the interaction length with the mean interparticle spacing, as nearly all do) and the ability to investigate the history of Lagrangian fluid element properties. Deformable grids (Gnedin 1995; Pen 1995; Xu 1995) and adaptive mesh refinement (Bryan & Norman 1995) significantly improve the spatial resolution of high-density regions in Eulerian codes.

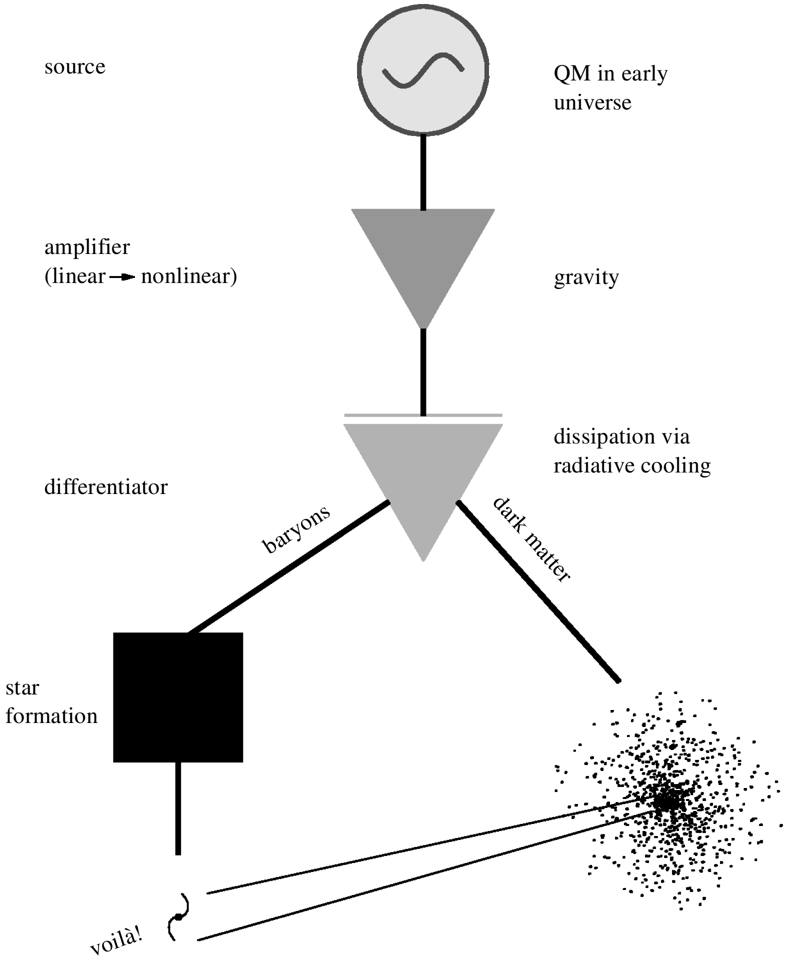

Figure 1. Schematic of the galaxy-formation process.

For any scheme, the total number of floating point operations $N_{\mathrm{op,tot}}$ sets a fundamental limit to what can be achieved by a given calculation. For an idealized scheme in which N_{el} elements (mesh points or particles) are evolved over N_{step} time-steps with N_{op} operations per step, the total operation count is the product $N_{\mathrm{step}}N_{\mathrm{op}}N_{\mathrm{el}}$. Choices made by the simulator determine how these cycles are burned. Modelling the gravitational dynamics of the dark matter alone minimizes N_{op} and focuses attention on the trade-off between N_{step} and N_{el}. The dynamic range in density is the controlling factor in this case. A moderate resolution study can minimize N_{step} and maximize N_{el}, at least up to the limit imposed by the total system memory. The billion-particle Hubble volume simulations of the Virgo Consortium (Evrard *et al.* 2000) are an example of this direction in parameter space. An alternative is to use large N_{el} to increase the dynamic range in mass density ρ within a fixed volume, but this entails an increase in N_{step} since the characteristic gravitational time scales as $\rho^{-1/2}$. The multi-million particle halos of Moore *et al.* (1999) typify this approach.

Coupling gas dynamics to gravity increases the operation count per step $N_{\rm op}$. The increase is minimized by a single-fluid treatment for the gas with no thermodynamic processes beyond shock heating and adiabatic heating/cooling. Adding a star formation and feedback prescription increases $N_{\rm op}$, perhaps dramatically so if multiple chemical species are considered and a non-equilibrium chemical network is employed to model radiative cooling processes in detail (Cen 1992; Anninos & Norman 1996; Abel *et al*. 2000). Additional physical complications such as magnetic fields, dust grains, and radiation transport have so far been largely ignored or treated in an approximate manner (Gnedin & Ostriker 1997). But developments of full three-dimensional radiation transport are underway.

The bottom line is that finite computational resources impose limits on what can be achieved by a particular computational project. Roughly speaking, one has to choose how far to proceed in the orthogonal directions of enhanced physics, enlarged dynamic range and improved statistics (number of objects in simulated ensemble). Examples discussed below illustrate some of the practical implications of this choice.

3. Gravity and simple gas dynamics

Certain ‘peripheral’ problems avoid explicit modelling of the galaxy-formation process but still provide information on the process by examining its boundary conditions. Galaxy clusters and the Lyα forest are prime examples of structure above and beneath that of galaxies. In the former, the dynamics of what might be termed ‘galactic surrogates’ and the structure of the intracluster medium (ICM) are areas of active investigation. In the latter, the view of the Lyα forest as a tracer of mildly nonlinear density fluctuations has provided a new approach to measuring the dark matter power spectrum on galactic scales.

(*a*) *Galaxy clusters: N-body*

Galaxy clusters offer a self-contained environment within which to study galaxy evolution. A longstanding issue that remains open is the extent to which the kinematic state of galaxies in clusters represents that of the underlying dark matter distribution. At stake are the accuracy of cosmological parameter inferences based on cluster galaxies (Borgani *et al*. 1997; Carlberg *et al*. 1997) as well as the general issue of how galaxies trace the dark matter in nonlinear regions of the Universe. Optical observations of clusters reveal regularity in the spatial distribution of cluster galaxies (West *et al*. 1987; Carlberg *et al*. 1997) that is likely to reflect the regularity of the underlying dark matter distribution, but perhaps to a biased degree.

Early attempts to use gas dynamic simulations with radiative cooling but without star formation resulted in a spatially compact bright galaxy population that implied potentially large systematic underestimates of cluster virial mass (Katz & White 1993; Evrard *et al*. 1994). This large inferred bias was subsequently shown to be an artefact of the viscous nature of the galaxies being modelled as clumps of cold gas particles. By converting gas in galaxies into collisionless ‘star’ particles through an instantaneous burst prior to cluster collapse, Frenk *et al*. (1996) produced a distribution of cluster galaxies that resembled much more closely that of the dark matter. Explicit modelling of star formation within galaxies and their feedback into the intracluster medium gas is the next step, and models of this type are under development.

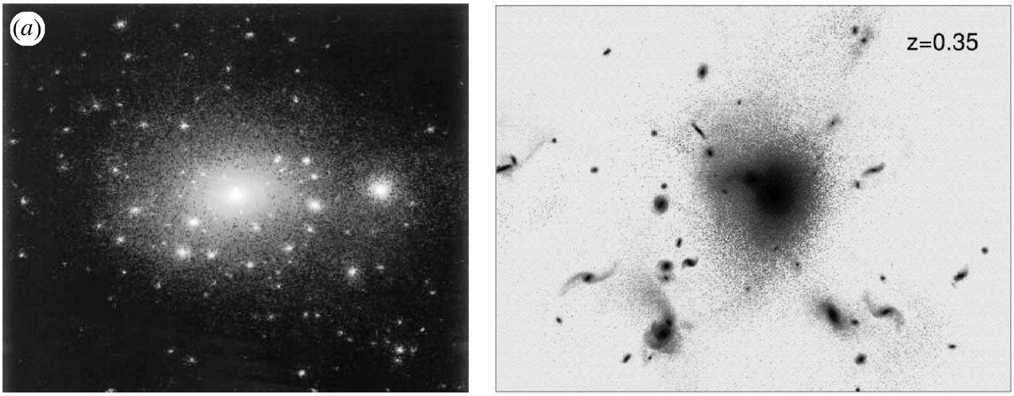

Figure 2. Examples of the detail afforded by very high-resolution N-body experiments. (a) The 'sub-halo' population resolved within the virial region (scale of the image) of a multi-million particle galaxy cluster simulation by Moore *et al.* (1999). (b) The central galaxy and surroundings from a multi-million particle simulation of Dubinski (1998).

Recent advances have been made with pure N-body approaches. Moore *et al.* (1999) realized a single cluster with over one million particles within its virial region (canonically defined as a sphere within which the mean density is roughly 200 times the background density) and identified a large population of bound, dark 'sub-halos' within the cluster's virial envelope (see figure 2a). These sub-halos are the tidally truncated cores of galactic-scale objects, remnants of the precursors that participated in forming the cluster through hierarchical merging. By using the Tully–Fisher relation to convert optical luminosities of galaxies in the Virgo cluster to circular velocity v_c, Moore *et al.* show that the simulated sub-halo abundance as a function of v_c matches well the observational data. Whether or not a simple one-to-one correspondence between sub-halo and galaxy will be preserved once gas processes are explicitly included remains to be seen. It is the pure N-body approximation that provides the wide dynamic range required to resolve the sub-halo population. Affording the CPU cycles to include gas physics would require a trade-off either in dynamic range or in the size of the system modelled. Reproducing a similar cluster with a full gas dynamic treatment simply awaits more processing power.

Another N-body approach to the problem of galaxy dynamics in clusters is to evolve a set of pre-formed galaxies through the process of merging to form a giant cluster. Dubinski (1998) takes this approach and investigates the roles that major galactic mergers and smaller galactic 'cannibalism' play in building a central dominant galaxy. His approach replaces galactic-scale halos at $z = 3$ with 'pre-formed' self-consistent disc + bulge + halo objects. The largest 20 objects are each resolved by 100 000 particles and 80 more are modelled using 10 000 particles each. The central galaxy forms through the merger of several massive galaxies along a filament early in the cluster's history (see figure 2b) and its final state displays no noticeable cD envelope. Its 'light' (meaning star particle) profile follows an $r^{1/4}$-law out to $200h^{-1}$ kpc. Gonzalez *et al.* (2000) report detection of a similar (this time real!) light profile of the central galaxy in A1651, from which they infer a formation history for this object similar to that of its virtual cousin.

(*b*) *Galaxy clusters: gas dynamics*

In the era of the ROSAT satellite, it was realized that the mass of the intracluster medium dominates that of galaxies in rich clusters. In Coma, the best studied local example, the mass ratio $M_{\mathrm{ICM}}/M_{\mathrm{gal}} = (10 \pm 2)(h/0.65)^{-3/2}$ is inferred from X-ray and optical observations (White *et al*. 1993). Because of the large thermal energy budget of the ICM, approaching 10^{64} erg for a cluster as massive as Coma, a reasonable expectation is that heating from galactic winds is a minor contributor compared with heating from shocks induced by gravitational collapse. In modelling terms, this motivates a computationally efficient treatment of clusters as dark matter gravitationally coupled to a single-phase gas that is subject to shock heating but no other heating or cooling processes. This simple treatment is surprisingly successful in reproducing the observed X-ray properties of massive clusters, but it was realized early on that this success did not extend to poorer clusters, those with X-ray temperatures $T_{\mathrm{X}} \lesssim 4\,\mathrm{keV}$ (Navarro *et al*. 1995). The ICM in these systems is more extended and less X-ray luminous than the simple models predict. Including galactic winds in the models, via a 'pre-heated' assumption for the gas (Evrard 1990; Navarro *et al*. 1995; Pen 1999) or via use of an explicit population of galaxies driving winds (Metzler & Evrard 1994), has led to success in reproducing the luminosity–temperature and isophotal size–temperature relations of clusters. However, the uniqueness and self-consistency of this solution remain to be addressed. In particular, the distribution of cluster metals, spatial density and temperature variations, and the evolution of the ICM properties with redshift are all features for which new data will soon be available from the *Chandra* and XMM/NEWTON observatories. Simulations that explicitly model galaxy formation and wind generation, needed to address these observations in detail, are under development.

(*c*) *Absorption line systems*

Along with the ICM in rich clusters, another successful application of gas dynamic simulations has been to model hydrogen absorption line systems observed in spectra of high redshift quasars (Cen *et al*. 1994; Zhang *et al*. 1995; Hernquist *et al*. 1996). Low-column-density 'forest' lines, once thought to be due to material in pressure-confined clouds either in galactic halos or intergalactic space, are now considered to arise from relatively low-density contrast HI embedded in the large-scale 'cosmic web' of the dark mass distribution. Statistical properties of the $z \sim 2$–3 forest lines are reproduced well by popular cosmological models after tuning of a single free parameter related to the ambient ionizing flux (see figure 3). In the models, the absorbing gas is only mildly overdense or even underdense with respect to the cosmic mean and it tracks well the structure of the dark matter. Under reasonable assumptions about its thermal history, it is described by a relatively simple equation of state (Bi & Davidsen 1997; Hui *et al*. 1997). The combination of these properties along with the cosmological principle of isotropy allows recovery of the dark matter power spectrum of density fluctuations $P(k)$ from the one-dimensional Fourier transform of the observed flux (Croft *et al*. 1999). A power-law fit derived from moderate resolution spectra—the first direct estimates of the linear mass power spectrum on these scales—yields an effective spectral index $n \equiv \mathrm{d}\ln P(k)/\mathrm{d}\ln k = -2.25 \pm 0.18$ on comoving scales of *ca.* 1–$10h^{-1}$ Mpc. This value agrees with expectations of cold dark matter (CDM) models in a variety of flavours. Coupling this measurement

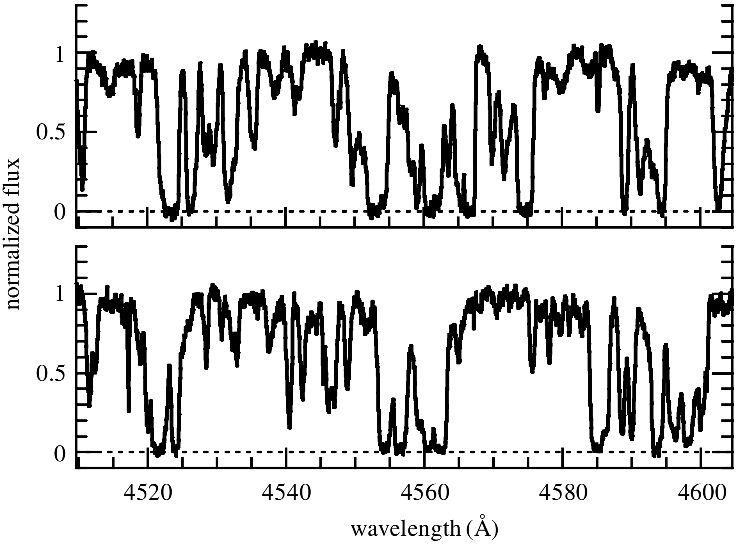

Figure 3. An absorption line spectrum of a quasi-stellar object (QSO) at redshift $z = 3.62$ along with a synthetic spectrum generated from a simulation of a universe dominated by vacuum energy. Which is real and which is virtual? (Figure courtesy of R. Davé and D. Weinberg; Keck data from Songaila & Cowie (1996).) Answer key and other examples of such real-virtual pairings can be found in Evrard (1999).

with COBE DMR† measurements of large-scale fluctuations and constraints from the abundance of galaxy clusters tends to select the currently popular Λ-dominated Universe (Philips *et al*. 2000).

At this meeting the results of Schaye *et al*. (2000) were presented. They employ simulations to calibrate measurement of the thermal history of the intergalactic medium (IGM) from the joint distribution of column density and line widths derived from Voigt profile decomposition of the Lyα forest. Preliminary analysis of a set of eight Keck HIRES spectra in the redshift range 2.0–4.5 indicates a jump in the temperature of the IGM between redshifts 2.5 and 3.5. The jump appears to be consistent with the heating expected from the second reionization of helium (HeII → HeIII).

4. Cosmological volumes: how do galaxies trace the mass?

The traditional path to the matter power spectrum, and thereby to information on the nature and amount of the still mysterious dark matter, is via the large-scale spatial distribution of galaxies. Current wide-angle spectroscopic galaxy surveys—2dF (http://www.mso.anu.edu.au/2dFGRS/) and SDSS (http://www.sdss.org/)—will soon return structural and kinematic data on comoving scales spanning from that sampled by the Lyα forest through to that probed by degree-scale microwave background anisotropies. Placing constraints on dark matter from these data will require an understanding of how the density and velocity fields of dark matter and galaxies are related.

---

† Differential Microwave Radiometer.

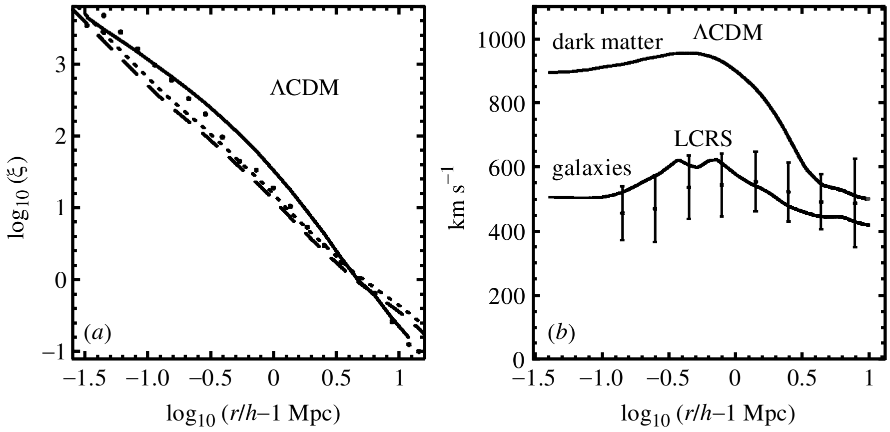

Figure 4. (*a*) Two-point correlation functions for simulated galaxies (filled circles) and dark matter (bold line) in a ΛCDM model. Estimates of the observed galaxy correlation function (Baugh 1996) are shown as dashed and dotted lines. (*b*) Pairwise velocities as a function of pair separation for simulated galaxies, real galaxies from the LCRS survey (dots) galaxies and dark matter.

A number of simulations have attempted to answer this question by including radiative cooling and possibly a star formation prescription within fairly large-scale cosmological volumes. The target for these simulations is a scale-dependent 'bias factor' $b(r)$ defined by the ratio of two-point correlation functions of dark matter (derived from straight N-body simulations) and galaxies, $b^2(r) \equiv \xi_{\mathrm{gal}}(r)/\xi_{\mathrm{DM}}(r)$, expectations of which vary depending on cosmology (Jenkins *et al.* 1998). In high-mass-density models $\Omega_{\mathrm{m}} = 1$, galaxies should be more clustered than the mass on scales 0.1–1h^{-1} Mpc, while the opposite is true for low-density models $\Omega_{\mathrm{m}} = 0.3$.

Biased galaxy populations are inferred from Eulerian simulations. Strauss *et al.* (1998) compare a one-dimensional velocity statistic for galaxies and dark matter and find a significantly cooler galaxy population in high density regions. Blanton *et al.* (1999) examine the scale dependence of the spatial bias for galaxies in a universe with mass density $\Omega_{\mathrm{m}} = 0.37$ and cosmological constant $\Omega_{\Lambda} = 0.63$. They find the bias to increase with decreasing scale, from values near unity at $30h^{-1}$ Mpc to *ca.* 2.5 at $1h^{-1}$ Mpc.

A recent Lagrangian calculation of nearly the same cosmology finds similar velocity bias, but different spatial bias, for the galaxy population. Jenkins *et al.* (2000) study a sample of roughly 2000 galaxies (resolved by 32 or more particles) arising in a four-million-particle calculation of a 100 Mpc periodic cube. The spatial and velocity correlations, shown in figure 4, reveal that the simulated galaxies are less clustered and cooler than the mass on scales between 150 and $3h^{-1}$ Mpc. Both of these biases make the virtual galaxy population a better match to observations compared with the dark matter.

The robustness of this beneficial outcome is challenged by the fairly large discrepancy in the spatial bias between the Eulerian and Lagrangian approaches to the same cosmology. The discrepancy may reflect a combination of different levels of spatial resolution (the Lagrangian gas dynamics is about a factor 10 more resolved in linear scale) and the different galaxy identification schemes employed. The underlying differences may not be understood until a comparison is performed among codes that have evolved the same initial conditions. Such an approach has established that simulations of the ICM in X-ray clusters have converged at the *ca.* 10% level (Frenk *et al.* 1999).

5. Galactic structure

Due to space limitations, I omit here detailed discussion about attempts to resolve the structure of stars and gas within galaxies. The reader can consult the recent review by Steinmetz (2000) for a perspective that raises challenges for popular cosmological models based on attempts to reproduce the Tully–Fisher relation of disc galaxies.

6. Summary

Direct simulation of the formation and large-scale clustering evolution of galaxies is evolving into a mature field. Advances in our understanding of galaxy formation will come from improved algorithms incorporating essential physics at deeper levels, from faster and larger computers, and, above all, from homogeneous, high quality and complete observations of galaxies, absorption line systems and galaxy clusters at a wide range of redshift.

References

Abel, T., Bryan, G. L. & Norman, M. L. 2000 *Astrophys. J.* (In the press.)

Anninos, P. & Norman, M. L. 1996 *Astrophys. J.* **459**, 12.

Baugh, C. M. 1996 *Mon. Not. R. Astr. Soc.* **280**, 267.

Bertschinger, E. 1998 *A. Rev. Astron. Astrophys.* **36**, 599.

Bi, H. G. & Davidsen, A. 1997 *Astrophys. J.* **479**, 523.

Blanton, M., Cen, R., Ostriker, J. P. & Strauss, M. 1999 *Astrophys. J.* **522**, 590.

Borgani, S., Gardini, A., Girardi, M. & Gottlober, S. 1997 *New Astron.* **2**, 119.

Bryan, G. L. & Norman, M. L. 1999 *BAAS* **187**, 9504.

Carlberg, R. G. (and 11 others) 1997 *Astrophys. J.* **485**, L13.

Cen, R. 1992 *Astrophys. J. Suppl.* **78**, 341.

Cen, R., Miralda-Escudé, J., Ostriker, J. P. & Rauch, M. 1994 *Astrophys. J.* **437**, L9.

Croft, R. A. C., Weinberg, D. H., Pettini, M., Katz, N. & Hernquist, L. 1999 *Astrophys. J.* **520**, 1.

Dubinski, J. 1998 *Astrophys. J.* **502**, 141.

Evrard, A. E. 1990 *Astrophys. J.* **363**, 349.

Evrard, A. E. 1999 *Proc. Natn. Acad. Soc.* **96**, 4228.

Evrard, A. E., Summers, F. J. & Davis, M. 1994 *Astrophys. J.* **422**, 11.

Evrard, A. E. *et al.* 2000 (In preparation.)

Frenk, C. S., Evrard, A. E., White, S. D. M. & Summers, F. J. 1996 *Astrophys. J.* **472**, 460.

Frenk, C. S. *et al.* 1999 *Astrophys. J.* **525**, 554.

Gnedin, N. Y. 1995 *Astrophys. J. Suppl.* **97**, 231.

Gnedin, N. Y. & Ostriker, J. P. 1997 *Astrophys. J.* **486**, 581.

Gonzalez, A. H., Zabludoff, A. I., Zaritsky, D. & Dalcanton, J. J. 2000 (In preparation.)

Hernquist, L., Katz, N., Weinberg, D. H. & Miralda-Escudé, J. 1996 *Astrophys. J.* **457**, L5.

Hui, L., Gnedin, N. & Zhang, Y. 1997 *Astrophys. J.* **486**, 599.

Jenkins, A. *et al*. 1998 *Astrophys. J.* **499**, 20.

Jenkins, A. *et al*. 2000 (In preparation.)

Katz, N. 1992 *Astrophys. J.* **391**, 502.

Katz, N. & White, S. D. M. 1993 *Astrophys. J.* **412**, 455.

Katz, N., Hernquist, L. & Weinberg, D. H. 1992 *Astrophys. J.* **399**, L100.

Larson, R. B. 1974 *Mon. Not. R. Astr. Soc.* **166**, 585.

Metzler, C. A. & Evrard, A. E. 1994 *Astrophys. J.* **437**, 564.

Moore, B., Ghigna, S., Governato, F., Lake, G., Quinn, T., Stadel, J. & Tozzi, P. 1999 *Astrophys. J.* **524**, L19.

Navarro, J. F. & White, S. D. M. 1994 *Mon. Not. R. Astr. Soc.* **267**, 401.

Navarro, J. F., Frenk, C. S. & White, S. D. M. 1995 *Mon. Not. R. Astr. Soc.* **275**, 720.

Pen, U.-L. 1995 *Astrophys. J. Suppl.* **100**, 269.

Pen, U. 1999 *Astrophys. J.* **510**, L1.

Philips, J., Weinberg, D. H., Croft, R. A. C., Hernquist, L., Katz, N. & Pettini, M. 2000 (In preparation.)

Schaye, J., Theuns, T., Rauch, M., Efstathiou, G. & Sargent, W. L. 2000 (In preparation.)

Songaila, A. & Cowie, L. L. 1996 *Astr. J.* **112**, 335.

Steinmetz, M. 2000 In *Galaxy dynamics: from the early Universe to the present* (ed. F. Combes, G. Mamon & V. Charmandari). (In the press.)

Strauss, M. A., Ostriker, J. P. & Cen, R. 1998 *Astrophys. J.* **494**, 20.

West, M. J., Dekel, A. & Oemler Jr, A. 1987 *Astrophys. J.* **316**, 1.

White, S. D. M., Navarro, J. F., Evrard, A. E. & Frenk, C. S. 1993 *Nature* **366**, 429.

Xu, G. 1995 *Astrophys. J. Suppl.* **98**, 355.

Zhang, Y., Anninos, P. & Norman, M. L. 1995 *Astrophys. J.* **453**, L57.