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Virtual galaxy formation

August E. Evrard

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By August E. Evrard**

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In this contribution, I briefly review the status of efforts to directly simulate the In this contribution, I briefly review the status of efforts to directly simulate the evolution of galaxies and their pre- and post-formation environments. A few phe-
nomenological applications are discussed; the intraclus In this contribution, I briefly review the status of efforts to directly simulate the evolution of galaxies and their pre- and post-formation environments. A few phenomenological applications are discussed: the intraclust evolution of galaxies and their pre- and post-formation environments. A few phenomenological applications are discussed: the intracluster medium in galaxy clusters, the structure of the Ly α forest, galaxy clustering in nomenological applications are discussed: the intracluster medium in galaxy clus-
ters, the structure of the Ly α forest, galaxy clustering in cosmic volumes and the
internal structure of galaxies. Given the strongly em ters, the structure of the $Ly\alpha$ forest, galaxy clustering in cosmic volumes and the internal structure of galaxies. Given the strongly empirical character of the problem, 'reverse engineering' of the observed galaxy dist internal structure of galaxies. Given the strongly empirical character of the problem,
'reverse engineering' of the observed galaxy distribution is a fair characterization of
the near-term state of affairs. The thorny prob "reverse engineering" of the observed galaxy distribution is a fair characterization of the near-term state of affairs. The thorny problem of how galaxies trace the overall mass on large scales may be yielding some ground the near-term state of affairs. The thorny problem of how galaxies trace the overall mass on large scales may be yielding some ground, but efforts to resolve smaller-scale features of galaxies are challenged by the complex kiloparsec scales.

Keywords: cosmology; galaxy formation; num erical simulation

1. Introduction

Modelling galaxy formation in a cosmic environment is a formidable task for a number of reasons. For starters, there is the problem of dynamic range. The character-
ber of reasons. For starters, there is the problem of dynamic range. The character-
istic half-light radii of bright galaxies are three or Modelling galaxy formation in a cosmic environment is a formidable task for a num-
ber of reasons. For starters, there is the problem of dynamic range. The character-
istic half-light radii of bright galaxies are three or ber of reasons. For starters, there is the problem of dynamic range. The characteristic half-light radii of bright galaxies are three orders of magnitude smaller than their mean separation, implying a range of 10^9 in v istic half-light radii of bright galaxies are three orders of magnitude smaller than
their mean separation, implying a range of 10^9 in volume. A similarly large factor
separates the stellar mass of an L_* galaxy from separates the stellar mass of an L_* galaxy from the mass of the smallest star form-
ing region within it. Second, the complexity of the physical processes driving the separates the stellar mass of an L_* galaxy from the mass of the smallest star forming region within it. Second, the complexity of the physical processes driving the conversion between gas and stars makes it difficult to ing region within it. Second, the complexity of the physical processes driving the conversion between gas and stars makes it difficult to justify a particular *a priori* description employed in numerical modelling. Finall conversion between gas and stars makes it difficult to justify a description employed in numerical modelling. Finally, the pro
three-dimensional; there are no spatial symmetries to exploit.
These factors are good excuses f scription employed in numerical modelling. Finally, the problem is intrinsically
ree-dimensional; there are no spatial symmetries to exploit.
These factors are good excuses for pessimism, but nature has kindly provided
wit

three-dimensional; there are no spatial symmetries to exploit.
These factors are good excuses for pessimism, but nature has kindly provided
us with reasons to be optimistic by giving us galaxy populations with remarkable
s These factors are good excuses for pessimism, but nature has kindly provided
us with reasons to be optimistic by giving us galaxy populations with remarkable
structural regularities. There is hope, then, to reverse enginee us with reasons to be optimistic by giving us galaxy populations with remarkable
structural regularities. There is hope, then, to reverse engineer the process. The mor-
phological classes of the Hubble sequence, the Tully– structural regularities. There is hope, then, to reverse engineer the process. The morphological classes of the Hubble sequence, the Tully–Fisher relation for disc galaxies, the fundamental plane of elliptical galaxies, an phological classes of the Hubble sequence, the Tully–Fisher relation for disc galaxies,
the fundamental plane of elliptical galaxies, and the morphology–density relation in
clusters imply the existence of organizing princi the fundamental plane of elliptical galaxies, and the morphology-density relation in
clusters imply the existence of organizing principles that we can hope to reproduce
by modelling what might be termed 'mesoscopic' phenom clusters imply the existence of organizing principles that we can hope to reproduce
by modelling what might be termed 'mesoscopic' phenomena. Essentially, the prob-
lem boils down to identifying one or more 'recipes' for t by modelling what might be termed 'mesoscopic' phenomena. Essentially, the prob-
lem boils down to identifying one or more 'recipes' for the star formation rate as a
function of local conditions and for the feedback, in te Olem boils down to identifying one or more 'recipes' for the star formation rate as a
function of local conditions and for the feedback, in terms of thermal, chemical and
kinetic returns, which that star formation has on i

So-called semi-analytic methods (SAMs), reviewed by Cole (this issue), offer an alternative approach to direct dynamical modelling. SAMs offer the ability to explore kinetic returns, which that star formation has on its local environment.
So-called semi-analytic methods (SAMs), reviewed by Cole (this issue), offer an
alternative approach to direct dynamical modelling. SAMs offer the ab So-called semi-analytic methods (SAMs), reviewed by Cole (this issue), offer an alternative approach to direct dynamical modelling. SAMs offer the ability to explore the controlling parameter space far more quickly than wo alternative approach to direct dynamical modelling. SAMs offer the ability to explore
the controlling parameter space far more quickly than would be possible via direct
simulation, but the approximations made to describe t the controlling parameter space far more quickly than would be possible via direct simulation, but the approximations made to describe the dynamical behaviour of the galaxies can be a liability for these methods, to a degr the galaxies can be a liability for these methods, to a degree depending on the
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particular issues under investigation. Clearly, the way forward is to combine both **MATHEMATICAL,
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approaches. Full dynamical modelling can assess the accuracy of approximations
employed by SAMs and the SAMs, in turn, can provide best est approaches. Full dynamical modelling can assess the accuracy of approximations approaches. Full dynamical modelling can assess the accuracy of approximemployed by SAMs and the SAMs, in turn, can provide best estimate prescriptor star formation and feedback to be used in the full dynamical modelling.

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2. Engineering galaxies: processes and limitations

In a universe with matter content dominated by cold dark matter, the basic physical
ingredients for galaxy formation outlined in figure 1 are now familiar. Gravity is, of
course, the key player. The growth of density fluct In a universe with matter content dominated by cold dark matter, the basic physical In a universe with matter content dominated by cold dark matter, the basic physical
ingredients for galaxy formation outlined in figure 1 are now familiar. Gravity is, of
course, the key player. The growth of density fluct ingredients for galaxy formation outlined in figure 1 are now familiar. Gravity is, of course, the key player. The growth of density fluctuations into the nonlinear regime leads to the emergence of a hierarchically evolvin course, the key player. The growth of density fluctuations into the nonlinear regime
leads to the emergence of a hierarchically evolving cosmic web of collapsed, dark
matter halos. Within the evolving potential well of a g leads to the emergence of a hierarchically evolving cosmic web of collapsed, dark
matter halos. Within the evolving potential well of a given halo, the baryonic matter
is heated by shocks but the optically thin gas can rap matter halos. Within the evolving potential well of a given halo, the baryonic matter
is heated by shocks but the optically thin gas can rapidly lose its thermal pressure
support via radiative cooling. The baryons then sin is heated by shocks but the optically thin gas can rapidly lose its thermal pressure
support via radiative cooling. The baryons then sink toward the halo core and, once
self-gravitating, suffer fragmentation instabilities support via radiative cooling. The baryons then sink toward the halo core and, once
self-gravitating, suffer fragmentation instabilities that produce local star forming
regions. The star formation process is represented as self-gravitating, suffer fragmentation instabilities that produce local star forming
regions. The star formation process is represented as a black box in figure 1 because
it is practically impossible to describe from first regions. The star formation process is represented as a black box in figure 1 because
it is practically impossible to describe from first principles. Instead, one employs
an empirically or heuristically motivated prescript it is practically impossible to describe from first principles. Instead, one employs
an empirically or heuristically motivated prescription. For example, a formulation
commonly used since Larson (1974) assumes that a fixe an empirically or heuristically motivated prescription. For exam
commonly used since Larson (1974) assumes that a fixed fraction
in star formation over a local dynamical time $\tau_{\rm dyn} \propto (G\rho)^{-1/2}$: $\overline{0}$:

$$
\frac{\mathrm{d}\rho_*}{\mathrm{d}t} = c_* \frac{\rho_{\text{gas}}}{\tau_{\text{dyn}}}.\tag{2.1}
$$

Modelling feedback from star formation is accomplished by introducing a stellar mass
function and calculating the mass and energy return over time from stellar winds
and supernovae. An important and presently uncertain iss Modelling feedback from star formation is accomplished by introducing a stellar mass Modelling feedback from star formation is accomplished by introducing a stellar mass
function and calculating the mass and energy return over time from stellar winds
and supernovae. An important and presently uncertain iss function and calculating the mass and energy return over time from stellar winds
and supernovae. An important and presently uncertain issue is the question of the
relative amounts of thermal and kinetic energy returned fr and supernovae. An important and presently uncertain issue is the question of the relative amounts of thermal and kinetic energy returned from star formation (Katz 1992; Navarro & White 1994). Since feedback is believed to relative amounts of thermal and kinetic energy returned from star formation (Katz 1992; Navarro $\&$ White 1994). Since feedback is believed to be a dominant player only in the smallest halos, its effects are often neglec **MATHEMATICAL,
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SCIENCES** 1992; Navarro & White 1994). Since feedback is believed to be a dominant player only in the smallest halos, its effects are often neglected in simulations designed to model the formation of bright galaxies in large cosmic model the formation of bright galaxies in large cosmic volumes (Katz *et al.* 1992; model the formation of bright galaxies in large cosmic volumes (Katz *et al.* 1992; Evrard *et al.* 1994). The lack of small-scale resolution in these experiments serves to mimic feedback, at least in the sense that gas c Evrard *et al.* 1994)
to mimic feedback,
unresolved scales.
Given prescription mimic feedback, at least in the sense that gas cannot be converted into stars on
resolved scales.
Given prescriptions for star formation processes, the equations describing the
mbination of ingredients in figure 1 constitu

unresolved scales.
Given prescriptions for star formation processes, the equations describing the
combination of ingredients in figure 1 constitute a well-posed initial-value prob-
lem that can be solved within a finite do Given prescriptions for star formation processes, the equations describing the combination of ingredients in figure 1 constitute a well-posed initial-value problem that can be solved within a finite domain with appropriat combination of ingredients in figure 1 constitute a well-posed initial-value prob-
lem that can be solved within a finite domain with appropriate boundary condi-
tions. Bertschinger (1998) presents the governing equations lem that can be solved within a finite domain with appropriate boundary conditions. Bertschinger (1998) presents the governing equations (see also Cen 1992) and reviews various choices of numerical methods. Eulerian (decom tions. Bertschinger (1998) presents the governing equations (see also Cen 1992) and
reviews various choices of numerical methods. Eulerian (decomposition by volume)
and Lagrangian (decomposition by mass) approaches offer c reviews various choices of numerical methods. Eulerian (decomposition by volume)
and Lagrangian (decomposition by mass) approaches offer complementary advan-
tages. Broadly speaking, advantages of Eulerian schemes are bett and Lagrangian (decomposition by mass) approaches offer complementary advantages. Broadly speaking, advantages of Eulerian schemes are better shock capturing, calculable truncation error, and equal spatial resolution of lo tages. Broadly speaking, advantages of Eulerian schemes are better shock capturing, calculable truncation error, and equal spatial resolution of low- and high-density regions. Advantages of Lagrangian schemes are automatic calculable truncation error, and equal spatial resolution of low- and high-density regions. Advantages of Lagrangian schemes are automatic advection of fluid properties, enhanced spatial resolution in high-density regions regions. Advantages of Lagrangian schemes are automatic advection of fluid properties, enhanced spatial resolution in high-density regions (for schemes that scale
the interaction length with the mean interparticle spacing, erties, enhanced spatial resolution in high-density regions (for schemes that scale
the interaction length with the mean interparticle spacing, as nearly all do) and the
ability to investigate the history of Lagrangian fl the interaction length with the mean interparticle spacing, as nearly all do) and the ability to investigate the history of Lagrangian fluid element properties. Deformable grids (Gnedin 1995; Pen 1995; Xu 1995) and adaptiv ability to investigate the history of Lagrangian fluid element properties. Deformable
grids (Gnedin 1995; Pen 1995; Xu 1995) and adaptive mesh refinement (Bryan &
Norman 1995) significantly improve the spatial resolution o grids (Gnedin 19
Norman 1995) sig
Eulerian codes. *Phil. Trans. R. Soc. Lond.* A (2000)

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Figure 1. Schematic of the galaxy-formation process.

Figure 1. Schematic of the galaxy-formation process.
For any scheme, the total number of floating point operations $N_{\text{op,tot}}$ sets a funda-
ental limit to what can be achieved by a given calculation. For an idealized sch For any scheme, the total number of floating point operations $N_{\text{op,tot}}$ sets a fundamental limit to what can be achieved by a given calculation. For an idealized scheme
in which N, elements (mesh points or particles) ar For any scheme, the total number of floating point operations $N_{\text{op,tot}}$ sets a funda-
mental limit to what can be achieved by a given calculation. For an idealized scheme
in which N_{el} elements (mesh points or part mental limit to what can be achieved by a given calculation. For an idealized scheme
in which $N_{\rm el}$ elements (mesh points or particles) are evolved over $N_{\rm step}$ time-steps
with $N_{\rm op}$ operations per step, the total with N_{op} operations per step, the total operation count is the product $N_{\text{step}}N_{\text{op}}N_{\text{el}}$.
Choices made by the simulator determine how these cycles are burned. Modelling the gravitational dynamics of the dark m tion on the trade-off between $N_{\rm step}$ and $N_{\rm el}$. The dynamic range in density is the gravitational dynamics of the dark matter alone minimizes N_{op} and focuses attention on the trade-off between N_{step} and N_{el} . The dynamic range in density is the controlling factor in this case. A moderate r tion on the trade-off between N_{step} and N_{el} . The dynamic range in density is the controlling factor in this case. A moderate resolution study can minimize N_{step} and maximize N_{el} , at least up to the lim controlling factor in this case. A moderate resolution study can minimize N_{step} and maximize N_{el} , at least up to the limit imposed by the total system memory. The billion-particle Hubble volume simulations of th billion-particle Hubble volume simulations of the Virgo Consortium (Evrard *et al.* 2000) are an example of this direction in parameter space. An alternative is to use large N_{el} to increase the dynamic range in mass de 2000) are an example of this direction in parameter space. An alternative is to use large N_{el} to increase the dynamic range in mass density ρ within a fixed volume, but this entails an increase in N_{step} since $\rho^{-1/2}$. The multi-million particle halos of Moore *et al.* (1999) typify this approach. N_{el} to increase the dynamic range in mass density ρ within a fixed volume, but mails an increase in N_{step} since the characteristic gravitational time scales as . The multi-million particle halos of Moore *et*

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Coupling gas dynamics to gravity increases the operation count per step N_{op} .
The increase is minimized by a single-fluid treatment for the gas with no thermodynamic processes beyond shock heating and adiabatic heating/cooling. Adding a The increase is minimized by a single-fluid treatment for the gas with no thermo-
dynamic processes beyond shock heating and adiabatic heating/cooling. Adding a
star formation and feedback prescription increases N_{op} dynamic processes beyond shock heating and adiabatic heating/cooling. Adding a
star formation and feedback prescription increases N_{op} , perhaps dramatically so if
multiple chemical species are considered and a non-eq star formation and feedback prescription increases N_{op} , perhaps dramatically so if
multiple chemical species are considered and a non-equilibrium chemical network
is employed to model radiative cooling processes in multiple chemical species are considered and a non-equilibrium chemical network
is employed to model radiative cooling processes in detail (Cen 1992; Anninos &
Norman 1996; Abel *et al.* 2000). Additional physical complica

is employed to model radiative cooling processes in detail (Cen 1992; Anninos & Norman 1996; Abel *et al.* 2000). Additional physical complications such as magnetic fields, dust grains, and radiation transport have so far Norman 1996; Abel *et al.* 2000). Additional physical complications such as magnetic fields, dust grains, and radiation transport have so far been largely ignored or treated in an approximate manner (Gnedin & Ostriker 1997 treated in an approximate manner (Gnedin & Ostriker 1997). But developments of full three-dimensional radiation transport are underway.

The bottom line is that finite computational resources impose limits on what can be achieved by a particular computational project. Roughly speaking, one has to The bottom line is that finite computational resources impose limits on what can
be achieved by a particular computational project. Roughly speaking, one has to
choose how far to proceed in the orthogonal directions of enh be achieved by a particular computational project. Roughly speaking, one has to choose how far to proceed in the orthogonal directions of enhanced physics, enlarged dynamic range and improved statistics (number of objects choose how far to proceed in the orthogonal directions of enhanced physics, enlarged
dynamic range and improved statistics (number of objects in simulated ensemble).
Examples discussed below illustrate some of the practica Examples discussed below illustrate some of the practical implications of this choice.
3. Gravity and simple gas dynamics

Certain `peripheral' problems avoid explicit modelling of the galaxy-formation process but still provide information on the process by examining its boundary con-Certain 'peripheral' problems avoid explicit modelling of the galaxy-formation process but still provide information on the process by examining its boundary conditions. Galaxy clusters and the Ly α forest are prime exa cess but still provide information on the process by examining its boundary conditions. Galaxy clusters and the $Ly\alpha$ forest are prime examples of structure above and beneath that of galaxies. In the former, the dynamics ditions. Galaxy clusters and the $Ly\alpha$ forest are prime examples of structure above
and beneath that of galaxies. In the former, the dynamics of what might be termed
'galactic surrogates' and the structure of the intraclu and beneath that of galaxies. In the former, the dynamics of what might be termed

'galactic surrogates' and the structure of the intracluster medium (ICM) are areas

of active investigation. In the latter, the view of th 'galactic surrogates' and the structure of the intracluster medium (ICM) are areas matter power spectrum on galactic scales.

(*a*) *Galaxy clusters:* ^N*-body*

Galaxy clusters offer a self-contained environment within which to study galaxy evolution. A longstanding issue that remains open is the extent to which the kinematic state of galaxies in clusters represents that of the underlying dark matter evolution. A longstanding issue that remains open is the extent to which the kinematic state of galaxies in clusters represents that of the underlying dark matter distribution. At stake are the accuracy of cosmological pa matic state of galaxies in clusters represents that of the underlying dark matter distribution. At stake are the accuracy of cosmological parameter inferences based on cluster galaxies (Borgani *et al.* 1997; Carlberg *et* distribution. At stake are the accuracy of cosmological parameter inferences based
on cluster galaxies (Borgani *et al.* 1997; Carlberg *et al.* 1997) as well as the general
issue of how galaxies trace the dark matter in n on cluster galaxies (Borgani *et al.* 1997; Carlberg *et al.* 1997) as well as the general issue of how galaxies trace the dark matter in nonlinear regions of the Universe. Optical observations of clusters reveal regulari issue of how galaxies trace the dark matter in nonlinear regions of the Universe.
Optical observations of clusters reveal regularity in the spatial distribution of cluster
galaxies (West *et al.* 1987; Carlberg *et al.* 19 Optical observations of clusters reveal regularity in the spatial distribution of galaxies (West *et al.* 1987; Carlberg *et al.* 1997) that is likely to reflect the reg of the underlying dark matter distribution, but per laxies (West *et al.* 1987; Carlberg *et al.* 1997) that is likely to reflect the regularity the underlying dark matter distribution, but perhaps to a biased degree.
Early attempts to use gas dynamic simulations with radi

of the underlying dark matter distribution, but perhaps to a biased degree.
Early attempts to use gas dynamic simulations with radiative cooling but without
star formation resulted in a spatially compact bright galaxy pop Early attempts to use gas dynamic simulations with radiative cooling but without
star formation resulted in a spatially compact bright galaxy population that implied
potentially large systematic underestimates of cluster star formation resulted in a spatially compact bright galaxy population that implied
potentially large systematic underestimates of cluster virial mass (Katz & White
1993; Evrard *et al.* 1994). This large inferred bias wa potentially large systematic underestimates of cluster virial mass (Katz & White 1993; Evrard *et al.* 1994). This large inferred bias was subsequently shown to be an artefact of the viscous nature of the galaxies being m 1993; Evrard *et al.* 1994). This large inferred bias was subsequently shown to be an artefact of the viscous nature of the galaxies being modelled as clumps of cold gas particles. By converting gas in galaxies into colli an artefact of the viscous nature of the galaxies being modelled as clumps of cold
gas particles. By converting gas in galaxies into collisionless 'star' particles through
an instantaneous burst prior to cluster collapse, gas particles. By converting gas in galaxies into collisionless 'star' particles through
an instantaneous burst prior to cluster collapse, Frenk *et al.* (1996) produced a
distribution of cluster galaxies that resembled m an instantaneous burst prior to cluster collapse, Frenk *et al.* (1996) produced a distribution of cluster galaxies that resembled much more closely that of the dark matter. Explicit modelling of star formation within gal distribution of cluster galaxies that resembled much more closely that of the dark development.

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Figure 2. Examples of the detail afforded by very high-resolution N-body experiments. (a) The

"sub-halo" population resolved within the virial region (scale of the image) of a multi-million Figure 2. Examples of the detail afforded by very high-resolution N-body experiments. (a) The 'sub-halo' population resolved within the virial region (scale of the image) of a multi-million particle galaxy cluster simulat Figure 2. Examples of the detail afforded by very high-resolution N -body experiments. (a) The 'sub-halo' population resolved within the virial region (scale of the image) of a multi-million particle galaxy cluster simul 'sub-halo' population resolved within the virial region (scale particle galaxy cluster simulation by Moore *et al.* (1999). (*b*) The from a multi-million particle simulation of Dubinski (1998).

Recent advances have been made with pure N-body approaches. Moore *et al.* (1999) realized a single cluster with over one million particles within its virial region (canonically defined as a sphere within which the mean de Recent advances have been made with pure N-body approaches. Moore *et al.* (1999) realized a single cluster with over one million particles within its virial region (canonically defined as a sphere within which the mean d (1999) realized a single cluster with over one million particles within its virial region (canonically defined as a sphere within which the mean density is roughly 200 times the background density) and identified a large (canonically defined as a sphere within which the mean density is roughly 200 times
the background density) and identified a large population of bound, dark 'sub-halos'
within the cluster's virial envelope (see figure 2a) the background density) and identified a large population of bound, dark 'sub-halos'
within the cluster's virial envelope (see figure 2a). These sub-halos are the tidally
truncated cores of galactic-scale objects, remnant within the cluster's virial envelope (see figure 2a). These sub-halos are the tidally truncated cores of galactic-scale objects, remnants of the precursors that participated in forming the cluster through hierarchical mer truncated cores of galactic-scale objects, remnants of the precursors that participated
in forming the cluster through hierarchical merging. By using the Tully–Fisher rela-
tion to convert optical luminosities of galaxies tion to convert optical luminosities of galaxies in the Virgo cluster to circular velocity v_c , Moore *et al*. show that the simulated sub-halo abundance as a function of v_c matches well the observational data. Whether **ICAL
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VCES** matches well the observational data. Whether or not a simple one-to-one corresponmatches well the observational data. Whether or not a simple one-to-one correspon-
dence between sub-halo and galaxy will be preserved once gas processes are explicitly
included remains to be seen. It is the pure N-body ap dence between sub-halo and galaxy will be preserved once gas processes are explicitly
included remains to be seen. It is the pure N -body approximation that provides the
wide dynamic range required to resolve the sub-hal included remains to be seen. It is the pure N -body approximation that provides the wide dynamic range required to resolve the sub-halo population. Affording the CPU cycles to include gas physics would require a trade-of wide dynamic range required to resolve the sub-halo population. Affording the CPU
cycles to include gas physics would require a trade-off either in dynamic range or
in the size of the system modelled. Reproducing a similar in the size of the system modelled. Reproducing a similar cluster with a full gas dynamic treatment simply awaits more processing power.

Another ^N-body approach to the problem of galaxy dynamics in clusters is to evolve a set of pre-formed galaxies through the process of merging to form a giant Another *N*-body approach to the problem of galaxy dynamics in clusters is to evolve a set of pre-formed galaxies through the process of merging to form a giant cluster. Dubinski (1998) takes this approach and investigate evolve a set of pre-formed galaxies through the process of merging to form a giant
cluster. Dubinski (1998) takes this approach and investigates the roles that major
galactic mergers and smaller galactic 'cannibalism' pla galactic mergers and smaller galactic 'cannibalism' play in building a central dom-G inant galaxy. His approach replaces galactic-scale halos at $z = 3$ with 'pre-formed'

Self-consistent disc + bulge + halo objects. The largest 20 objects are each resolved

So by 100 000 particles and 80 more are modell self-consistent disc $+$ bulge $+$ halo objects. The largest 20 objects are each resolved self-consistent disc + bulge + halo objects. The largest 20 objects are each resolved
by 100 000 particles and 80 more are modelled using 10 000 particles each. The cen-
tral galaxy forms through the merger of several mas by 100 000 particles and 80 more are modelled using 10 000 particles each. The central galaxy forms through the merger of several massive galaxies along a filament early in the cluster's history (see figure 2b) and its fi tral galaxy forms through the merger of several massive galaxies along
early in the cluster's history (see figure 2b) and its final state displays
able cD envelope. Its 'light' (meaning star particle) profile follows an $1/4$ -la a filament
no notice-
law out to
real!) light early in the cluster's history (see figure 2b) and its final state displays no notice-
able cD envelope. Its 'light' (meaning star particle) profile follows an $r^{1/4}$ -law out to
 $200h^{-1}$ kpc. Gonzalez *et al.* (2000) r able cD envelope. Its 'light' (meaning star particle) profile follows an $r^{1/4}$ -law out to $200h^{-1}$ kpc. Gonzalez *et al.* (2000) report detection of a similar (this time real!) light profile of the central galaxy in A $200h^{-1}$ kpc. Gonzalez *et al.* (2000) report detection of a similar (this time real!) light profile of the central galaxy in A1651, from which they infer a formation history for this object similar to that of its virtua

(*b*) *Galaxy clusters: gas dynamics*

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SCIENCES In the era of the ROSAT satellite, it was realized that the mass of the intraclus-
ter medium dominates that of galaxies in rich clusters. In Coma, the best studied
local example, the mass ratio $M_{\text{ICM}}/M_{\text{cal}} = (10 + 2)(h$ In the era of the ROSAT satellite, it was realized that the mass of the intracluster medium dominates that of galaxies in rich clusters. In Coma, the best studied local example, the mass ratio $M_{\rm ICM}/M_{\rm gal} = (10 \pm 2)(h/0.6$ In the era of the ROSAT satellite, it was realized that the mass of the intracluster medium dominates that of galaxies in rich clusters. In Coma, the best studied ter medium dominates that of galaxies in rich clusters. In Coma, the best studied
local example, the mass ratio $M_{\text{ICM}}/M_{\text{gal}} = (10 \pm 2)(h/0.65)^{-3/2}$ is inferred from
X-ray and optical observations (White *et al.* 1993)

local example, the mass ratio $M_{\text{ICM}}/M_{\text{gal}} = (10 \pm 2)(h/0.65)^{-3/2}$ is inferred from X-ray and optical observations (White *et al.* 1993). Because of the large thermal energy budget of the ICM, approaching 10^{64} erg X-ray and optical observations (White *et al.* 1993). Because of the large thermal energy budget of the ICM, approaching 10^{64} erg for a cluster as massive as Coma, a reasonable expectation is that heating from galacti energy budget of the ICM, approaching 10^{64} erg for a cluster as massive as Coma,
a reasonable expectation is that heating from galactic winds is a minor contributor
compared with heating from shocks induced by gravita a reasonable expectation is that heating from galactic winds is a minor contributor
compared with heating from shocks induced by gravitational collapse. In modelling
terms, this motivates a computationally efficient treatm compared with heating from shocks induced by gravitational collapse. In modelling
terms, this motivates a computationally efficient treatment of clusters as dark matter
gravitationally coupled to a single-phase gas that is terms, this motivates a computationally efficient treatment of clusters as dark matter
gravitationally coupled to a single-phase gas that is subject to shock heating but no
other heating or cooling processes. This simple t gravitationally coupled to a single-phase gas that is subject to shock heating but no
other heating or cooling processes. This simple treatment is surprisingly successful
in reproducing the observed X-ray properties of mas other heating or cooling processes. This simple treatment is surprisingly successful in reproducing the observed X-ray properties of massive clusters, but it was realized
early on that this success did not extend to poorer clusters, those with X-ray temper-
atures $T_{\rm X} \le 4$ keV (Navarro *et al.* 1995). early on that this success did not extend to poorer clusters, those with X-ray temperatures $T_X \leq 4$ keV (Navarro *et al.* 1995). The ICM in these systems is more extended and less X-ray luminous than the simple models p atures $T_{\rm X} \lesssim 4$ keV (Navarro *et al.* 1995). The ICM in these systems is more extended
and less X-ray luminous than the simple models predict. Including galactic winds in
the models, via a 'pre-heated' assumption fo the models, via a 'pre-heated' assumption for the gas (Evrard 1990; Navarro *et al.* 1995; Pen 1999) or via use of an explicit population of galaxies driving winds (Metzler $\&$ Evrard 1994), has led to success in reprodu 1995; Pen 1999) or via use of an explicit population of galaxies driving winds (Metzler $\&$ Evrard 1994), has led to success in reproducing the luminosity-temperature and isophotal size-temperature relations of clusters. & Evrard 1994), has led to success in reproducing the luminosity-temperature and
isophotal size-temperature relations of clusters. However, the uniqueness and self-
consistency of this solution remain to be addressed. In p isophotal size–temperature relations of clusters. However, the uniqueness and self-
consistency of this solution remain to be addressed. In particular, the distribution of
cluster metals, spatial density and temperature va consistency of this solution remain to be addressed. In particular, the distribution of cluster metals, spatial density and temperature variations, and the evolution of the ICM properties with redshift are all features for cluster metals, spatial density and temperature variations, and the evolution of the ICM properties with redshift are all features for which new data will soon be available from the *Chandra* and XMM/NEWTON observatories. ICM properties with redshift are all features for which new data will soon be available
from the *Chandra* and XMM/NEWTON observatories. Simulations that explicitly
model galaxy formation and wind generation, needed to add from the *Chandra* and XMM/NEW model galaxy formation and wind in detail, are under development. (*c*) *Absorption line systems*

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SCIENCES** Along with the ICM in rich clusters, another successful application of gas dynamic simulations has been to model hydrogen absorption line systems observed in spectra simulations has been to model hydrogen absorption line systems observed in spectra
of high redshift quasars (Cen *et al.* 1994; Zhang *et al.* 1995; Hernquist *et al.* 1996).
Low-column-density 'forest' lines, once though of high redshift quasars (Cen *et al*. 1994; Zhang *et al*. 1995; Hernquist *et al*. 1996). of high redshift quasars (Cen *et al.* 1994; Zhang *et al.* 1995; Hernquist *et al.* 1996).
Low-column-density 'forest' lines, once thought to be due to material in pressure-
confined clouds either in galactic halos or int Low-column-density 'forest' lines, once thought to be due to material in pressure-
confined clouds either in galactic halos or intergalactic space, are now considered
to arise from relatively low-density contrast HI embed confined clouds either in galactic halos or intergalactic space, are now considered
to arise from relatively low-density contrast HI embedded in the large-scale 'cos-
mic web' of the dark mass distribution. Statistical pr lines are reproduced well by popular cosmological models after tuning of a single lines are reproduced well by popular cosmological models after tuning of a single
free parameter related to the ambient ionizing flux (see figure 3). In the models,
the absorbing gas is only mildly overdense or even underd \blacktriangleright free parameter related to the ambient ionizing flux (see figure 3). In the models, free parameter related to the ambient ionizing flux (see figure 3). In the models,
the absorbing gas is only mildly overdense or even underdense with respect to the
cosmic mean and it tracks well the structure of the dark the absorbing gas is only mildly overdense or even underdense with respect to the cosmic mean and it tracks well the structure of the dark matter. Under reasonable assumptions about its thermal history, it is described by cosmic mean and it tracks well the structure of the dark matter. Under reasonable assumptions about its thermal history, it is described by a relatively simple equation of state (Bi & Davidsen 1997; Hui *et al.* 1997). Th assumptions about its thermal history, it is described by a relatively simple equation
of state (Bi & Davidsen 1997; Hui *et al.* 1997). The combination of these properties
along with the cosmological principle of isotrop of state (Bi & Davidsen 1997; Hui *et al.* 1997). The combination of these properties
along with the cosmological principle of isotropy allows recovery of the dark matter
power spectrum of density fluctuations $P(k)$ from along with the cosmological principle of isotropy allows recovery of the dark matter
power spectrum of density fluctuations $P(k)$ from the one-dimensional Fourier trans-
form of the observed flux (Croft *et al.* 1999). A \bullet power spectrum of density fluctuations $P(k)$ from the one-dimensional Fourier trans-
form of the observed flux (Croft *et al.* 1999). A power-law fit derived from moderate
resolution spectra—the first direct estimates of form of the observed flux (Croft *et al.* 1999). A power-law fit derived from moderate resolution spectra—the first direct estimates of the linear mass power spectrum on these scales—yields an effective spectral index $n \$ resolution spectra—the first direct estimates of the linear mass power spectrum on
these scales—yields an effective spectral index $n \equiv d\ln P(k)/d\ln k = -2.25 \pm 0.18$
on comoving scales of *ca*. 1–10h⁻¹ Mpc. This value agrees these scales—yields an effective spectral index $n \equiv \text{d} \ln P(k)/\text{d} \ln k = -2.25 \pm 0.18$
on comoving scales of *ca*. 1-10*h*⁻¹ Mpc. This value agrees with expectations of cold
dark matter (CDM) models in a variety of flavou dark matter (CDM) models in a variety of flavours. Coupling this measurement *Phil. Trans. R. Soc. Lond.* A (2000)

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wavelength (A)
Figure 3. An absorption line spectrum of a quasi-stellar object (QSO) at redshift $z = 3.62$ along
with a synthetic spectrum generated from a simulation of a universe dominated by vacuum Figure 3. An absorption line spectrum of a quasi-stellar object (QSO) at redshift $z = 3.62$ along with a synthetic spectrum generated from a simulation of a universe dominated by vacuum energy. Which is real and which is ō with a synthetic spectrum generated from a simulation of a universe dominated by vacuum
energy. Which is real and which is virtual? (Figure courtesy of R. Davé and D. Weinberg; Keck with a synthetic spectrum generated from a simulation of a universe dominated by vacuum
energy. Which is real and which is virtual? (Figure courtesy of R. Davé and D. Weinberg; Keck
data from Songaila & Cowie (1996).) Answ energy. Which is real and which i
data from Songaila & Cowie (1996)
can be found in Evrard (1999).

can be found in Evrard (1999).
with COBE DMR[†] measurements of large-scale fluctuations and constraints from with COBE DMR[†] measurements of large-scale fluctuations and constraints from
the abundance of galaxy clusters tends to select the currently popular Λ -dominated
Universe (Philips *et al.* 2000) with COBE DMR[†] measureme:
the abundance of galaxy cluster
Universe (Philips *et al.* 2000).
At this meeting the results of e abundance of galaxy clusters tends to select the currently popular Λ -dominated
niverse (Philips *et al.* 2000).
At this meeting the results of Schaye *et al.* (2000) were presented. They employ
nulations to calibrate

Universe (Philips *et al.* 2000).
At this meeting the results of Schaye *et al.* (2000) were presented. They employ
simulations to calibrate measurement of the thermal history of the intergalactic
medium (IGM) from the jo At this meeting the results of Schaye *et al.* (2000) were presented. They employ simulations to calibrate measurement of the thermal history of the intergalactic medium (IGM) from the joint distribution of column density simulations to calibrate measurement of the thermal history of the intergalactic
medium (IGM) from the joint distribution of column density and line widths derived
from Voigt profile decomposition of the Ly α forest. Pr medium (IGM) from the joint distribution of column density and line widths derived
from Voigt profile decomposition of the Ly α forest. Preliminary analysis of a set
of eight Keck HIRES spectra in the redshift range 2.0 from Voigt profile decomposition of the Ly α forest. Preliminary analysis of a set
of eight Keck HIRES spectra in the redshift range 2.0–4.5 indicates a jump in the
temperature of the IGM between redshifts 2.5 and 3.5. of eight Keck HIRES spectra in the redshift range 2.0–4.5 indicates a jump in the temperature of the IGM between redshifts 2.5 and 3.5. The jump appears to be consistent with the heating expected from the second reionizat

HeIII).

4. Cosmological volumes: how do galaxies trace the mass?

The traditional path to the matter power spectrum, and thereby to information on The traditional path to the matter power spectrum, and thereby to information on
the nature and amount of the still mysterious dark matter, is via the large-scale
spatial distribution of galaxies. Current wide-angle spectr The traditional path to the matter power spectrum, and thereby to information on
the nature and amount of the still mysterious dark matter, is via the large-scale
spatial distribution of galaxies. Current wide-angle spect the nature and amount of the still mysterious dark matter, is via the large-scale
spatial distribution of galaxies. Current wide-angle spectroscopic galaxy surveys—
 $2dF (http://www.mso.anu.edu.au/2dFGRS/)$ $2dF (http://www.mso.anu.edu.au/2dFGRS/)$ $2dF (http://www.mso.anu.edu.au/2dFGRS/)$ and SDSS (<http://www.sdss.org/>) spatial distribution of galaxies. Current wide-angle spectroscopic galaxy surveys— $2dF$ (http://www.mso.anu.edu.au/ $2dFGRS$) and SDSS (http://www.sdss.org/) will soon return structural and kinematic data on comoving scales spanning from
that sampled by the $Ly\alpha$ forest through to that probed by degree-scale microwave
background anisotropies. Placing constraints on dark matter that sampled by the Ly α forest through to that probed by degree-scale microwave
background anisotropies. Placing constraints on dark matter from these data will
require an understanding of how the density and velocity background anisotropies. Placing constraints on dark matter from these data will

† Differential Microwave Radiometer.

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 $log_{10}(r/h-1 \text{ Mpc})$ $log_{10}(r/h-1 \text{ Mpc})$
Figure 4. (a) Two-point correlation functions for simulated galaxies (filled circles) and dark
matter (bold line) in a ACDM model. Estimates of the observed galaxy correlation function Figure 4. (a) Two-point correlation functions for simulated galaxies (filled circles) and dark matter (bold line) in a Λ CDM model. Estimates of the observed galaxy correlation function (Baugh 1996) are shown as dashed matter (bold line) in a Λ CDM model. Estimates of the observed galaxy correlation function (Baugh 1996) are shown as dashed and dotted lines. (b) Pairwise velocities as a function of pair separation for simulated galaxies, real galaxies from the LCRS survey (dots) galaxies and dark matter.

A number of simulations have attempted to answer this question by including A number of simulations have attempted to answer this question by including radiative cooling and possibly a star formation prescription within fairly large-scale cosmological volumes. The target for these simulations is a A number of simulations have attempted to answer this question by including radiative cooling and possibly a star formation prescription within fairly large-scale cosmological volumes. The target for these simulations is cosmological volumes. The target for these simulations is a scale-dependent 'bias factor' $b(r)$ defined by the ratio of two-point correlation functions of dark matter cosmological volumes. The target for these simulations is a so factor' $b(r)$ defined by the ratio of two-point correlation funct (derived from straight N-body simulations) and galaxies, $b^2(r)$ expectations of which vary $2(r)$ factor' $b(r)$ defined by the ratio of two-point correlation functions of dark matter (derived from straight N-body simulations) and galaxies, $b^2(r) \equiv \xi_{gal}(r)/\xi_{DM}(r)$, expectations of which vary depending on cosmology (Jen (derived from straight N-body simulations) and galaxies, $b^2(r) \equiv \xi_{gal}(r)/\xi_{\text{DM}}(r)$,
expectations of which vary depending on cosmology (Jenkins *et al.* 1998). In high-
mass-density models $\Omega_{\text{m}} = 1$, galaxies should expectations of which vary depending on cosmology (Jenkins *et al.* 1998). In high mass-density models $\Omega_{\rm m} = 1$, galaxies should be more clustered than the mass or scales 0.1–1 h^{-1} Mpc, while the opposite is true fo

mass-density models $\Omega_m = 1$, galaxies should be more clustered than the mass on scales $0.1-1h^{-1}$ Mpc, while the opposite is true for low-density models $\Omega_m = 0.3$. Biased galaxy populations are inferred from Eulerian s **MATHEMATICAL,
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SCIENCES** Biased galaxy populations are inferred from Eulerian simulations. Strauss *et al.* (1998) compare a one-dimensional velocity statistic for galaxies and dark matter and find a significantly cooler galaxy population in high find a significantly cooler galaxy population in high density regions. Blanton *et al.* (1999) examine the scale dependence of the spatial bias for galaxies in a universe with mass density $\Omega_m = 0.37$ and cosmological con with mass density $\Omega_{\rm m} = 0.37$ and cosmological constant $\Omega_{\Lambda} = 0.63$. They find the bias to increase with decreasing scale, from values near unity at $30h^{-1}$ Mpc to *ca*. 2.5 at $1h^{-1}$ Mpc. with mass dens
bias to increase
at $1h^{-1}$ Mpc.
A recent Lagre as to increase with decreasing scale, from values near unity at $30h^{-1}$ Mpc to ca. 2.5
 $1h^{-1}$ Mpc.

A recent Lagrangian calculation of nearly the same cosmology finds similar velocity

as but different spatial bias for

at $1h^{-1}$ Mpc.
A recent Lagrangian calculation of nearly the same cosmology finds similar velocity
bias, but different spatial bias, for the galaxy population. Jenkins *et al.* (2000) study
a sample of roughly 2000 galax A recent Lagrangian calculation of nearly the same cosmology finds similar velocity
bias, but different spatial bias, for the galaxy population. Jenkins *et al.* (2000) study
a sample of roughly 2000 galaxies (resolved by a sample of roughly 2000 galaxies (resolved by 32 or more particles) arising in a four-million-particle calculation of a 100 Mpc periodic cube. The spatial and velocity four-million-particle calculation of a 100 Mpc periodic cube. The spatial and velocity
correlations, shown in figure 4, reveal that the simulated galaxies are less clustered
and cooler than the mass on scales between 150 correlations, shown in figure 4, reveal that the simulated galaxies are less clustered
and cooler than the mass on scales between 150 and $3h^{-1}$ Mpc. Both of these biases
make the virtual galaxy population a better match and cooler than the mass on scales between 150 and $3h^{-1}$ Mpc. Both of these biases
make the virtual galaxy population a better match to observations compared with
the dark matter.
The robustness of this beneficial outco \bullet make the virtual galaxy population a better match to observations compared with

the dark matter.
The robustness of this beneficial outcome is challenged by the fairly large discrep-
ancy in the spatial bias between the Eulerian and Lagrangian approaches to the same
cosmology. The discrepancy may refle The robustness of this beneficial outcome is challenged by the fairly large discrepancy in the spatial bias between the Eulerian and Lagrangian approaches to the same cosmology. The discrepancy may reflect a combination of ancy in the spatial bias between the Eulerian and Lagrangian approaches to the same
cosmology. The discrepancy may reflect a combination of different levels of spatial
resolution (the Lagrangian gas dynamics is about a fac cosmology. The discrepancy may reflect a combination of different levels of spatial resolution (the Lagrangian gas dynamics is about a factor 10 more resolved in linear scale) and the different galaxy identification scheme

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differences may not be understood until a comparison is performed among codes
that have evolved the same initial conditions. Such an approach has established that differences may not be understood until a comparison is performed among codes that have evolved the same initial conditions. Such an approach has established that simulations of the ICM in X-ray clusters have converged at differences may not be understood until a comparison is performed among codes
that have evolved the same initial conditions. Such an approach has established that
simulations of the ICM in X-ray clusters have converged at *ethat have evol* simulations of *et al.* 1999).

5. Galactic structure

Due to space limitations, I omit here detailed discussion about attempts to resolve the structure of stars and gas within galaxies. The reader can consult the recent review Due to space limitations, I omit here detailed discussion about attempts to resolve the
structure of stars and gas within galaxies. The reader can consult the recent review
by Steinmetz (2000) for a perspective that raises structure of stars and gas within galaxies. The reader can consult the recent review
by Steinmetz (2000) for a perspective that raises challenges for popular cosmological
models based on attempts to reproduce the Tully–Fis Impodels based on attempts to reproduce the Tully–Fisher relation of disc galaxies.

6. Summary

6. Summary
Direct simulation of the formation and large-scale clustering evolution of galaxies
is evolution into a mature field. Advances in our understanding of galaxy formation Direct simulation of the formation and large-scale clustering evolution of galaxies
is evolving into a mature field. Advances in our understanding of galaxy formation
will come from improved algorithms incorporating essent Direct simulation of the formation and large-scale clustering evolution of galaxies
is evolving into a mature field. Advances in our understanding of galaxy formation
will come from improved algorithms incorporating essent is evolving into a mature field. Advances in our understanding of galaxy formation will come from improved algorithms incorporating essential physics at deeper levels, from faster and larger computers, and, above all, from will come from improved algorithms incorporating essential physics at deeper levels, from faster and larger computers, and, above all, from homogeneous, high quality and complete observations of galaxies, absorption line s from faster and larger compi
and complete observations of
at a wide range of redshift.

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